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Tugas EAS Analisis Vektor

1). a). perlihatkan bahwa $\overline{F}(x,y,z) = y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}$ adalah medan vektor Konservatif !

Jawab : $\nabla \times \overline{F} = 0$ (cek medan vektor konservatif)

$$\nabla \times \overline{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix}$$

$$= (6xyz^2 - 6xyz^2)\hat{i} - (3y^2z^2 - 3y^2z^2)\hat{j} + (2yz^3 - 2yz^3)\hat{k}$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$= \mathbf{0}$$

Terbukti, bahwa $\overline{F}(x,y,z) = y^2z^3\hat{i} + 2xyz^3\hat{j} + 3xy^2z^2\hat{k}$ adalah Medan Vektor Konservatif

b). Carilah fungsi $f(x,y,z)$ sedemikian hingga $\overline{F}(x,y,z) = \overline{\nabla} f(x,y,z)$

$$\overline{F}(x,y,z) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) . f(x,y,z)$$

$$= \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right)$$

- $\frac{\partial f}{\partial x} = y^2z^3$
  $f = xy^2z^3 + C_1$ ... (1)
- $\frac{\partial f}{\partial y} = 2xyz^3$
  $f = xy^2z^3 + C_2$ ... (2)
- $\frac{\partial f}{\partial z} = 3xy^2z^2$
  $f = xy^2z^3 + C_3$ ... (3)

▶ Substitusi (1) ke (2)

$$xy^2z^3 + C_1 = xy^2z^3 + C_2$$
$$C_1 = C_2$$

▶ Substitusi (2) ke (3)

$$xy^2z^3 + C_2 = xy^2z^3 + C_3$$
$$C_2 = C_3$$

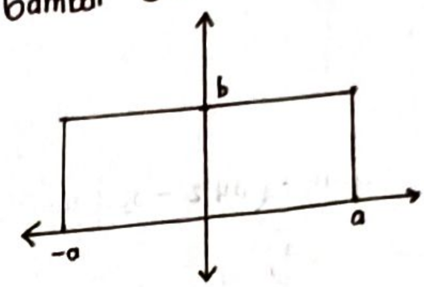
Sehingga, $C_1 = C_2 = C_3 = C$

Diperoleh, fungsi $f(x,y,z)$ yang memenuhi adalah

$$f(x,y,z) = xy^2z^3 + C$$

2). Tunjukkan kebenaran Teorema Green untuk $\overline{F}(x,y,z) = (x^2+y^2)\hat{i} - 2xy\hat{j}$ dimana A adalah daerah empat persegi panjang yang dibatasi oleh : $x = \pm a$; $y = 0$; dan $y = b$ (Hitung dengan Teorema Green dan secara langsung).

▶ Gambar Daerah Penyelesaian



▶ Perhitungan Langsung

- Garis $y = 0$

$$\int_C F.dr = \int_{-a}^{a} [(x^2+y^2)\hat{i} - 2xy\hat{j}](dx\hat{i} + 0\hat{j})$$
$$= \int_{-a}^{a} (x^2)\hat{i}\, dx$$
$$= \frac{1}{3}x^3 \Big|_{-a}^{a}$$
$$= \frac{1}{3}[a^3 - (-a)^3]$$
$$= \frac{2}{3}a^3$$

- Garis $x = a$

$$\int_C F.dr = \int_0^b [(x^2+y^2)\hat{i} - 2xy\hat{j}](0\hat{i} + dy\hat{j})$$
$$= \int_0^b -2ay\, dy$$
$$= -ay^2 \Big|_0^b$$
$$= -ab^2$$

- Garis $y = b$

$$\int_C F.dr = \int_a^{-a} [(x^2+y^2)\hat{i} - 2xy\hat{j}](dx\hat{i} + 0\hat{j})$$
$$= \int_a^{-a} x^2 + b^2\, dx$$
$$= \frac{1}{3}x^3 + b^2x \Big|_a^{-a}$$
$$= -\frac{a^3}{3} - ab^2 - \left(\frac{a^3}{3} + ab^2\right)$$

$$= -\frac{a^3}{3} - \frac{a^3}{3} - ab^2 - ab^2$$
$$= -\frac{2}{3}a^3 - 2ab^2$$

•> Garis $x = -a$

$$\int_c F.dr = \int_b^0 \left[(x^2+y^2)\hat{i} - 2xy\hat{j}\right]\left(0\hat{i} + dy\hat{j}\right)$$
$$= \int_b^0 -2xy\, dy$$
$$= \int_b^0 -2(-a)y\, dy$$
$$= 2a \cdot \frac{1}{2}y^2 \Big|_b^0$$
$$= a\left[0 - b^2\right]$$
$$= -ab^2$$

▶ Hasil Total dengan Perhitungan Langsung

$$\int_c F.dr = \frac{2}{3}a^3 - ab^2 - \frac{2}{3}a^3 - 2ab^2 - ab^2$$
$$= -4ab^2$$

▶ Perhitungan dengan Teorema Green

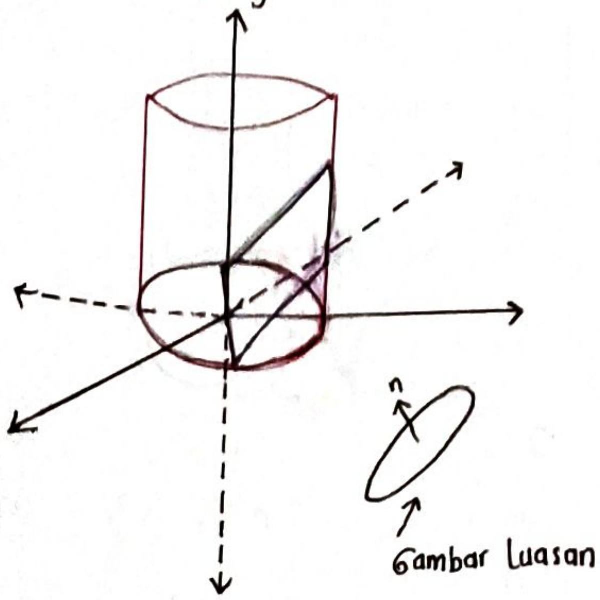
$$\iint_A \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dy\, dx$$
$$= \int_{-a}^{a}\int_0^b -2y - 2y \; dy\, dx$$
$$= -4\int_{-a}^{a} \frac{1}{2}y^2 \Big|_0^b dx$$
$$= -4\int_{-a}^{a} \frac{1}{2}b^2\, dx$$
$$= -2 \cdot b^2 x \Big|_{-a}^{a}$$
$$= -2b^2 (a - (-a))$$
$$= -4ab^2 \quad \text{(TERBUKTI)}$$

Jadi, Teorema Green terbukti berlaku dalam permasalahan ini.

3). Gunakan teorema Stokes untuk menghitung $\oint_c \bar{F} \cdot d\bar{r}$ dengan $\bar{F} = 2z\hat{i} + x\hat{j} + 3y\hat{k}$ dan C adalah batas luasan ellips bidang $z = x$ yang berada dalam silinder $x^2 + y^2 = 4$ yang berorientasi searah dengan putaran jarum jam seperti yang digambarkan jika dilihat dari atas.

Jawab :

▶ Gambar Bidang



$$\oint F.\, dr = \iint_S (\nabla \times F) \cdot n\, dS$$
$$= \iint_S \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z & x & 3y \end{vmatrix} (i - k)\, dS$$
$$= \iint_S \left[(3-0)\hat{i} - (0-2)\hat{j} + k\right](i - k)\, dS$$
$$= \iint_S \left[(3\hat{i} + 2\hat{j} + \hat{k})\right] \cdot (i - k)\, dS$$
$$= \iint_S (3-1)\, dS$$
$$= 2\iint_S dS$$

$L_{elips} = \pi ab$

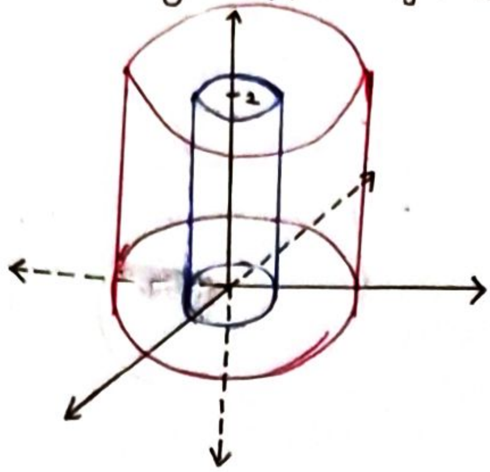
Diketahui sumbu minor elips $a = 2$ dan sumbu mayor elips $b = 2\sqrt{2}$, maka

$$\iint_S dS = \pi(2)(2\sqrt{2})$$
$$= 4\pi\sqrt{2}$$

$$\oint_c F.\, dr = 2(4\pi\sqrt{2})$$
$$= 8\pi\sqrt{2}$$

4). Diketahui medan vektor $\overline{F}(x,y,z) = 2z\hat{i} + x\hat{j} + z^2\hat{k}$

Dengan teorema Gauss, hitung $\iint \overline{F}.\overline{n}\, dS$ dengan S adalah permukaan benda padat berupa shell silinder (lihat gambar), $1 \le x^2+y^2 \le 4$, $0 \le z \le 2$



Jawab :

$$\iint_S F.n\, dK = \iiint_V \text{div } F\, dv$$

$$= \iint\int_0^2 2z\, dz\, dy\, dx$$

$$= 2\iint_A \frac{1}{2} z^2 \Big|_0^2 dy\, dx$$

$$= \iint_A 4\, dy\, dx$$

$$= 4.4\int_0^{\pi/2}\int_0^2 r\, dr\, d\theta$$

$$= 16\int_0^{\pi/2} \frac{1}{2} r^2 \Big|_0^2 d\theta$$

$$= 8\int_0^{\pi/2} 4\, d\theta$$

$$= 32.\ \theta\Big|_0^{\pi/2}$$

$$= 32.\frac{\pi}{2}$$

$$= 16\pi$$